International Aid from the Recipient Perspective: Public or Private Good Input Assistance?

December 14-15, 2017

Tatsuyoshi Miyakoshi, Hosei University
1. **Motivation and Purpose:** non-competitive & non-excludable; benefit across countries

The distribution of ODA has ten categories: The three major

I. **Social infrastructure** (incl. Educational bldg. and hospitals)
II. **Economic infrastructure** (incl. Air Port, Railways)
III. **Production Sectors** (incl. Chemical, Energy production facilities)

- Thailand, India, Brazil, Ukraine and their neighbor countries are achieving rapid economic growth and have received both *public good assistance* for economic infrastructure and *private good assistance* for production sectors.
- These types of assistance can both complement or substitute with each other in the production process.
- The donor countries are located in the North developed regions, geographically far from the recipient areas in the South.

Figure 1-a. Gross disbursements of ODA in recipient countries in 2012  Figure 1-b. Gross disbursements of ODA for donor countries in 2012

How should a donor country decide between *public good assistance and private good assistance*? The donors are now faced with this important issues with regard to ODA.
Survey of Recent Contribution in ODA Policy Researches:
Hatzipanayotou & Michael (1995), Vicary & Sandler (2002), Schweinberger & Lahiri (2006), Chong & Gradstein (2008) emphasize its public good characteristics from the donors’ perspective and from the interaction between the donor and recipient countries, while they focus mainly on social infrastructure as output in a utility function.

However,
(i) Recent ODA support for rapidly developing regions has been directed at economic infrastructure, which is public good input in the production function, and moreover has been faced with choices between public or private good input assistance.
(ii) A donor’s aid makes recipients weak in public good provision by inducing recipients to become free riders. However, previous studies have not dealt with free-rider problems.

We need a new theoretical framework for recent ODA support to developing countries.
(iii) Labor is an important input in production together with public and private good inputs.

Recently, the proportion of international migrants employed in the labor force has increased throughout the world, because of increasing economic openness and European integration.

Hatzipanayotou and Michael (2012) and Bandyopadhyay et al. (2014) have discussed the relations between migration, foreign aid, and the welfare state; however, they did not consider the choice between private goods input and public good input in the production product, which affects other countries’ production, as well as migration.

Thus, the previous papers are unrelated to such recent ODA support for the developing countries.
Purpose:

(i) construct a theoretical model from the recipient perspective that considers what type of assistance a donor country should provide: public good assistance or private good assistance.

(ii) test the model empirically for East Asia with the rapidly growing economies and a large ODA disbursement, by using the available data.

To our knowledge, no theoretical or empirical papers have examined public versus private good assistance. However, ODA is now facing this important issue.
2. Model. 2.1. Framework

Country i’s (=1,..,n) production: \( F_i(x_i, G) = x_i^{\alpha_i} G^{\beta_i} \); \( \alpha_i > 0, \beta_i > 0, 1 = \alpha_i + \beta_i \)

1. \( x_i \) is the quantity of the private good input and \( G \) is the total quantity of public good input.

Country i’s labor constraint \( L_i \): 

\[ L_i = s_i x_i + p_i g_i \]  \hspace{1cm} (1)

2. \( L_i > 0 \) is exogenously given, \( g_i \geq 0 \) is her contribution to the public good input and \( s_i, p_i > 0 \) is the labor input coefficient (1/productivity) for private and public good.

3. The total quantity of the public good \( G \) equals the sum of individual voluntary contributions \( g_i \), 

\[ G = \sum_{i=1}^{n} g_i \]

and the voluntary contributions by all except \( i \) is:

\[ G_{-i} = \sum_{j \neq i}^{n} g_j, \text{ then } G = G_{-i} + g_i \]

4. Under Nash assumption, country i maximizes the output in terms of \( x_i, g_i \), given \( G_{-i} \) and subject to labor constraint \( L_i > 0 \) as follows:

\[ \text{Max} \ x \ F_i(x_i, G_{-i} + g_i), \ s.t. \ L_i = s_i x_i + p_i g_i \]  \hspace{1cm} (2)
2.2. Replacement function

The Nash equilibrium solution is the set of \( \{x_1^*, ..., x_n^*, g_1^*, ..., g_n^*\} \) for \( i = 1, 2, ..., n: \)

\[
g_i^* = \hat{g}_i(G_{-i}) = \arg \max_{g_i} \left\{ x_i^\alpha, G_i^{\beta_i} = \left( \frac{L_i}{s_i - p_i g_i/s_i} \right)^{\alpha_i} (g_i + G_{-i})^{\beta_i} \mid g_i \geq 0, x_i \geq 0 \right\}
\]

(3)

The solution \( g_i^* \) can be rewritten

\[
g_i = \gamma_i(G) \equiv \begin{cases} \frac{L_i}{p_i} - \frac{\alpha_i}{\beta_i} G, & \text{if } \hat{w}_i > G \\ 0, & \text{otherwise} \end{cases}
\]

(4)

When only \{1,2,..,k\} of the \( n \) countries contribute to the public good, \( G_k^* = g_1^* + g_2^* + ... + g_k^* \) then because of (4), \( G_k^* \) is:

\[
G_k^* = \sum_{i=1}^{k} g_i^* = \sum_{i=1}^{k} \frac{L_i}{p_i} - G_k^* \sum_{i=1}^{k} \frac{\alpha_i}{\beta_i}, \quad \text{where } \hat{w}_i \equiv \beta_i L_i / p_i \alpha_i
\]

(5)

Using (4), (5) and the labor constraint in (1),

\[
g_i^* = \frac{L_i}{p_i} - \left( \frac{\sum_{i=1}^{k} L_i / p_i}{1 + \sum_{i=1}^{k} \alpha_i / \beta_i} \right) \frac{\alpha_i}{\beta_i} > 0 \quad x_i^* = \left( \frac{\sum_{i=1}^{k} L_i / p_i}{1 + \sum_{i=1}^{k} \alpha_i / \beta_i} \right) \frac{p_i \alpha_i}{s_i \beta_i}, \quad F_i(x_i^*, G_k^*)
\]

(6)

\[
g_i^* = 0, \quad x_i^* = L_i / s_i, \quad F_i(x_i^*, G_k^*) \quad \text{for non-contributors}
\]
\[ g^*_i = \hat{g}_i(G_{-i}) = \arg \max_{g_i} \left\{ \hat{x}^\alpha_i G^\beta_i = (L_i/s_i - p_i g_i/s_i)^\alpha_i (g_i + G_{-i})^\beta_i \mid g_i \geq 0, x_i \geq 0 \right\} \]

\[
\frac{\partial (x_i^\alpha_i G^\beta_i)}{\partial g_i} = \alpha_i x_i^{\alpha_i-1} (-p_i/s_i)(G)^{\beta_i} + x_i^{\alpha_i} \beta_i(G)^{\beta_i-1} = 0,
\]

\[
\alpha_i (p_i/s_i) = (L_i/s_i - p_i g_i/s_i) \beta_i(G)^{-1}, \quad s_i, \alpha_i, \beta_i G
\]

\[ g_i = L_i/p_i - (\alpha_i/\beta_i)G \]

\[
\frac{\partial^2 (x_i^\alpha_i G^\beta_i)}{\partial g_i^2} = \alpha_i (\alpha_i - 1) x_i^{\alpha_i-2} (-p_i/s_i)^2 (G)^{\beta_i} + \alpha_i x_i^{\alpha_i-1} (-p_i/s_i) \beta_i(G)^{\beta_i-1} + \alpha_i x_i^{\alpha_i-1} (-p_i/s_i) (G)^{\beta_i-1} + x_i^{\alpha_i} \beta_i (\beta_i - 1)(G)^{\beta_i-2} < 0
\]
3.1. Algorithm for contributors and Equilibrium

How do we search for contributor $k$ to decide $G^*_k$ in (5)? We apply Miyakoshi and Suzuki (2014, AE)’s algorithm to this problem:

Step 0: Assume the subscript $q$ is attached to $\hat{W}_i$ in descending order: $\hat{W}_n \leq \ldots \leq \hat{W}_{q+1} \leq \hat{W}_q \leq \ldots \leq \hat{W}_2 \leq \hat{W}_1$. Set $q = 1$.

Step 1: Solve $G^*_q$ in (5).

$$G^*_k = \sum_{i=1}^{k} g^*_i = \sum_{i=1}^{k} \frac{L_i}{p_i} - G^*_k \sum_{i=1}^{k} \frac{\alpha_i}{\beta_i}, \quad \text{hence } G^*_k = \frac{\sum_{i=1}^{k} \frac{L_i}{p_i}}{1 + \sum_{i=1}^{k} \frac{\alpha_i}{\beta_i}}$$  \hspace{1cm} \text{in (5)}

Step 2: Stop if the condition $(\hat{W}_{q+1} \leq G^*_q < \hat{W}_q)$ is satisfied. Otherwise $q = q + 1$ and repeat Step 1.

Understand visually and rationalize the algorithm of $k$ in Figure 1

$$g_i = \gamma_i(G) = \begin{cases} \frac{L_i}{p_i} - \frac{\alpha_i}{\beta_i}G, & \text{if } G < \hat{W}_i \\ 0, & \text{otherwise} \end{cases} \quad \text{where } \hat{W}_i = \beta_i L_i / p_i \alpha_i$$

$\gamma_1(G)$ $\gamma_2(G)$ $\gamma_3(G)$ $\gamma_4(G)$ $\gamma_5(G)$

Aggregate Replacement Function

Individual Replacement Function
3.2. Which assistance is better in terms of increasing production? :donor country’s price exogenous provision

Exogenous provision $t/p_0$ of public goods such highways and railways.

\[

g_i(G_{-i}) = \arg \max_{s_i} \left\{ \frac{L_i}{s_i} - \frac{\alpha_i (G + t/p_0)}{\beta_i} \mid 0 \leq g_i, 0 \leq x_i \right\}: \\
G^*_k + t/p_0 = \sum_{i=1}^{k} g_i^* + t/p_0 = \sum_{i=1}^{k} \frac{L_i}{p_i} - \frac{(g_i^* + t/p_0)}{\beta_i} + t/p_0, \\
g_i = \gamma_i(G) = \begin{cases}
\frac{L_i}{s_i}, & G < \hat{w}_i - t/p_0 \\
0, & \hat{w}_i - t/p_0 \geq G
\end{cases}
\tag{8}
\]

\[x^*_i = \frac{L_i}{s_i} - \frac{p_i g_i}{s_i \beta_i} \left( G^*_k + t/p_0 \right), \quad F_i(x^*_i, G^*_k + t/p_0) = \left( \frac{p_i \alpha_i}{s_i \beta_i} \right)^{\alpha_i} \left( G^*_k + t/p_0 \right) \tag{10}\]

Exogenous provision $t/s_0$ of private goods such machine among positive contributors:

\[
g_i(G_{-i}) = \arg \max_{s_i} \left\{ (x_i + \theta t/s_0) - \frac{\alpha_i (G + t/s_0)}{\beta_i} \mid 0 \leq g_i, 0 \leq x_i \right\}: \\
G^*_k = \sum_{i=1}^{k} g_i^* = \sum_{i=1}^{k} \frac{L_i + s_i \theta t/s_0}{p_i} - G^*_k \sum_{i=1}^{k} \frac{\alpha_i}{\beta_i}, \\
g_i = \gamma_i(G) = \begin{cases}
\frac{L_i + s_i \theta t/s_0}{p_i} - \frac{\alpha_i G}{\beta_i}, & G < \hat{w}_i + \frac{s_i \beta_i}{p_i \alpha_i} \theta t/s_0 \\
0, & \hat{w}_i + \frac{s_i \beta_i}{p_i \alpha_i} \theta t/s_0 \geq G
\end{cases} \tag{12}
\]

then \[G^{**}_k = \frac{(t/s_0) \sum_{i=1}^{k} s_i \theta t/s_0 + \sum_{i=1}^{k} L_i / p_i}{1 + \sum_{i=1}^{k} \alpha_i / \beta_i} \tag{13}\]

\[x^*_i = t/s_0 \theta t/s_0 = \frac{p_i \alpha_i}{s_i \beta_i} G^{**}_k, \quad F_i(x^*_i, \theta t/s_0, G^{**}_k) = \left( \frac{p_i \alpha_i}{s_i \beta_i} \right)^{\alpha_i} \left( G^{**}_k \right) \tag{14}\]

Which type of ODA is better?

\[
G^*_k + t/p_0 - G^{**}_k = \frac{t \left( \frac{1}{p_0} - \frac{1}{s_0} \sum_{i=1}^{k} \theta_i s_i / p_i \right)}{1 + \sum_{i=1}^{k} \alpha_i / \beta_i} > 0: \text{Public good assistance; } s_0 / p_0 > \sum_{i=1}^{k} \theta_i s_i / p_i
\]
Immigration Effects:

Immigration from positive contributor \( j \) to \( i \) with \( p_j > p_i \) in (9),(13):

\[
\frac{dG^*_k}{dL_i} = \left( \frac{1}{p_i} - \frac{1}{p_j} \right) > 0: \quad \text{For } i: \quad \frac{dg^*_i}{dL_i} > 0, \quad \frac{dx^*_i}{dL_i} > 0, \quad \frac{dF_i(x^*_i, G^*_k)}{dL_i} > 0, \quad \text{For } j: \quad \frac{dg^*_j}{dL_i} < 0, \quad \frac{dx^*_j}{dL_i} > 0, \quad \frac{dF_j(x^*_j, G^*_k)}{dL_i} > 0
\]

total public good input increases. Then output for all countries increase, as seen in (10) and (14). The effects are the same in both assistances.

Free-rider Effects:

Public good assistance: a free-rider
Private good assistance: a contributor
4. The Indochinese Peninsula as an Example

The Indochinese peninsula: Indonesia (INA), Laos (LAO), Malaysia (MA), Philippines (PH), Thailand (TH), and Vietnam (VT).
We need parameters: $\beta_i, p_i, s_i, L_i$

Table 1. Estimates of the Cobb–Douglas production using World Bank annual data.

$$
\log Y_t = \log A + \delta_1 DU_{1t} + \delta_2 DU_{2t} + \alpha \log x_t + \beta \log g_t : \quad A, a, \beta > 0, \quad \alpha + \beta = 1
$$

$$
DU_{1t} = \begin{cases} 
1 & \text{when } 1998:1 \leq t \leq 2007:1 \\
0 & \text{otherwise}
\end{cases}, \quad
DU_{2t} = \begin{cases} 
1 & \text{when } 2008:1 \leq t \\
0 & \text{otherwise}
\end{cases}
$$

Table 1. Estimated production function parameters

<table>
<thead>
<tr>
<th>Country i</th>
<th>$\log A$</th>
<th>$\delta_1$</th>
<th>$\delta_2$</th>
<th>$\alpha_i$</th>
<th>$\beta_i$</th>
<th>$\bar{R}^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>INA</td>
<td>0.26</td>
<td>0.18</td>
<td>0.74*</td>
<td>0.2702</td>
<td>0.7298*</td>
<td>0.87</td>
</tr>
<tr>
<td>1972–2012</td>
<td>(0.21)</td>
<td>(0.13)</td>
<td>(0.14)</td>
<td></td>
<td>(0.0626)</td>
<td></td>
</tr>
<tr>
<td>MA</td>
<td>−0.21</td>
<td>0.71*</td>
<td>1.16*</td>
<td>0.2362</td>
<td>0.7638*</td>
<td>0.83</td>
</tr>
<tr>
<td>1970–2012</td>
<td>(0.26)</td>
<td>(0.13)</td>
<td>(0.16)</td>
<td></td>
<td>(0.0550)</td>
<td></td>
</tr>
<tr>
<td>LAO</td>
<td>−0.36*</td>
<td>−0.18*</td>
<td>0.34*</td>
<td>0.1302</td>
<td>0.8698*</td>
<td>0.99</td>
</tr>
<tr>
<td>1988–2012</td>
<td>(0.11)</td>
<td>(0.05)</td>
<td>(0.06)</td>
<td></td>
<td>(0.0471)</td>
<td></td>
</tr>
<tr>
<td>PH</td>
<td>0.73*</td>
<td>0.69*</td>
<td>0.62*</td>
<td>0.2425</td>
<td>0.7575*</td>
<td>0.87</td>
</tr>
<tr>
<td>1970–2012</td>
<td>(0.32)</td>
<td>(0.21)</td>
<td>(0.24)</td>
<td></td>
<td>(0.0499)</td>
<td></td>
</tr>
<tr>
<td>TH</td>
<td>1.48*</td>
<td>−0.30*</td>
<td>0.01</td>
<td>0.3011</td>
<td>0.6989*</td>
<td>0.96</td>
</tr>
<tr>
<td>1975–2012</td>
<td>(0.18)</td>
<td>(0.10)</td>
<td>(0.11)</td>
<td></td>
<td>(0.0387)</td>
<td></td>
</tr>
<tr>
<td>VT</td>
<td>1.80*</td>
<td>0.18</td>
<td>−0.08</td>
<td>0.6209</td>
<td>0.3792#</td>
<td>0.26</td>
</tr>
<tr>
<td>1990–2012</td>
<td>(0.61)</td>
<td>(0.23)</td>
<td>(0.21)</td>
<td></td>
<td>(0.2425)</td>
<td></td>
</tr>
</tbody>
</table>

Notes: numbers in parentheses are standard deviations. * and # indicate significance at the 5% and 10% levels, respectively.
Table 2. Estimated Labor Force Coefficients

$L_t = C + d_1 DU_1 + d_2 DU_2 + s x_t + p g_t : s, p > 0$

<table>
<thead>
<tr>
<th>Country i</th>
<th>C</th>
<th>$d_1$</th>
<th>$d_2$</th>
<th>$s_i$</th>
<th>$p_i$</th>
<th>$\bar{R}^2$</th>
<th>$L_{2012-C-d_2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>INA</td>
<td>70.54*</td>
<td>17.81*</td>
<td>21.19*</td>
<td>0.0986#</td>
<td>0.0391*</td>
<td>0.91</td>
<td>26.65</td>
</tr>
<tr>
<td>1990–2012</td>
<td>(3.23)</td>
<td>(2.50)</td>
<td>(4.04)</td>
<td>(0.0899)</td>
<td>(0.0134)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MA</td>
<td>6.94*</td>
<td>2.45*</td>
<td>3.55*</td>
<td>0.0763#</td>
<td>0.0036</td>
<td>0.89</td>
<td>2.23</td>
</tr>
<tr>
<td>1990–2012</td>
<td>(0.64)</td>
<td>(0.28)</td>
<td>(0.48)</td>
<td>(0.0572)</td>
<td>(0.0044)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LAO</td>
<td>1.85*</td>
<td>0.22*</td>
<td>0.36*</td>
<td>0.7544*</td>
<td>0.0607*</td>
<td>0.97</td>
<td>1.11</td>
</tr>
<tr>
<td>1990–2012</td>
<td>(0.04)</td>
<td>(0.05)</td>
<td>(0.09)</td>
<td>(0.2284)</td>
<td>(0.0105)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PH</td>
<td>24.55*</td>
<td>6.79*</td>
<td>9.69*</td>
<td>0.5448#</td>
<td>0.0224*</td>
<td>0.88</td>
<td>7.04</td>
</tr>
<tr>
<td>1990–2012</td>
<td>(1.06)</td>
<td>(0.90)</td>
<td>(1.57)</td>
<td>(0.5040)</td>
<td>(0.0101)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TH</td>
<td>30.86*</td>
<td>2.41*</td>
<td>4.46*</td>
<td>0.2889*</td>
<td>0.0062#</td>
<td>0.87</td>
<td>4.1</td>
</tr>
<tr>
<td>1990–2012</td>
<td>(0.75)</td>
<td>(0.70)</td>
<td>(1.07)</td>
<td>(0.1404)</td>
<td>(0.0098)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>VT</td>
<td>32.86*</td>
<td>5.55*</td>
<td>2.33</td>
<td>0.7266*</td>
<td>0.0903*</td>
<td>0.96</td>
<td>17.67</td>
</tr>
<tr>
<td>1990–2012</td>
<td>(0.59)</td>
<td>(0.77)</td>
<td>(2.10)</td>
<td>(0.2426)</td>
<td>(0.0181)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: see notes in Table 1. $L_{2012}$ is the labor force in 2012, which is measured in millions: INA (118.38), MA (12.72), LAO (1.11), PH (7.04), TH (4.1), VT (17.67).
4.1. Three questions for the Indochinese Peninsula

(i) Who are the free riders? MA, INA, TH and PH are contributors. VET, LAO are free riders.

(ii) Which ODA policy is optimal? When the relative labor productivity \( \frac{S_i}{P_i} \) of producing the public good to that of the private good of a donor is larger than average relative productivity \( \sum_{i=1}^{k} \theta_i s_i / p_i \), the public good assistance is optimal. Notes that private good assistance only on contributors.

(iii) What are the effects of migration on the optimal ODA policy?
The migration from TH to MA produces a Pareto-improving.

(iv) Free-rider effect?
The larger public good assistance induces more free-riders, where the PH may be free-rider.

Table 3. Who are the free riders? Algorithm for \( k \)

<table>
<thead>
<tr>
<th>Country &amp; number</th>
<th>( \beta_i )</th>
<th>( s_i )</th>
<th>( p_i ) (million)</th>
<th>( \hat{w}_i ) = ( \beta_i L_i / p_i \alpha_i )</th>
<th>( G^*<em>k = \frac{\sum</em>{i=1}^{k} L_i / p_i}{1 + \sum_{i=1}^{k} \alpha_i / \beta_i} ) (thousand)</th>
<th>( \frac{s_i}{p_i} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>MA 1</td>
<td>0.7638</td>
<td>0.0763</td>
<td>0.0036</td>
<td>2.23</td>
<td>2003.098</td>
<td>473.132</td>
</tr>
<tr>
<td>INA 2</td>
<td>0.7298</td>
<td>0.0986</td>
<td>0.0391</td>
<td>26.65</td>
<td>1840.937</td>
<td>774.662</td>
</tr>
<tr>
<td>TH 3</td>
<td>0.6989</td>
<td>0.2889</td>
<td>0.0062</td>
<td>4.1</td>
<td>1534.958</td>
<td>929.877</td>
</tr>
<tr>
<td>PH 4</td>
<td>0.7575</td>
<td>0.5448</td>
<td>0.0224</td>
<td>7.04</td>
<td>981.7378</td>
<td>936.708</td>
</tr>
<tr>
<td>VT 5</td>
<td>0.3792</td>
<td>0.7266</td>
<td>0.0903</td>
<td>17.67</td>
<td>120.1356</td>
<td></td>
</tr>
<tr>
<td>LAO 6</td>
<td>0.8950</td>
<td>0.4944</td>
<td>0.0640</td>
<td>1.11</td>
<td>20.73063</td>
<td></td>
</tr>
</tbody>
</table>
16
References


Faini, R. and Venturini, A. (1993),” Trade, Aid and Migration: Some basic policy issues“, European Economic Review 37, 435-442


Appendix 1: Case (ii)

We assume that the set of contributors remains unchanged because the amount of assistance is appropriately small, now, suppose private good input $t$ is provided exogenously to both positive contributors and noncontributors with the ratio:

$$0 < \gamma_i < 1 \text{ and } 1 \equiv \sum_{i=1}^{k} \gamma_i + \sum_{i=k+1}^{n} \gamma_i;$$

$$\sum_{i=1}^{k} \gamma_i > 0 \text{ for contributors and } \sum_{i=k+1}^{n} \gamma_i > 0 \text{ for noncontributors}. \quad (A1)$$

Comparing the solution for (13), (14), and (15) where private good input $t$ is provided exogenously to only positive contributors, the only difference in the solution applies to noncontributors:

$$x_{i}^{**} = \frac{L_i}{s_i} + \gamma_i t, \quad F_i(x_{i}^{**}, G_k^{**}) \equiv \left(\frac{L_i}{s_i}\right)^{\alpha_i} \left(G_k^{**}\right)^{\beta_i}. \quad (A2)$$

Then, we focus only on the comparison of production levels for the noncontributors. When $t/p_0 > t/s_0 \sum_{i=1}^{k} \gamma_i s_i / p_i$ (i.e., $s_0 / p_0 > \sum_{i=1}^{k} \gamma_i s_i / p_i$) and $\gamma_i$ (for $i = k+1, \ldots, n$) is appropriately small, condition (A3) holds.

$$\frac{F_i(x_{i}^{**} + t/p_0, G_k)}{F_i(x_{i}^{**} + \gamma t/s_0, G_k)} = \left(\frac{L_i/s_i}{s_i + \gamma t/s_0}\right)^{\alpha_i} \left(\frac{\sum_{i=1}^{k} L_i/p_i + t/p_0}{\sum_{i=1}^{k} L_i/p_i + t/s_0 \sum_{i=1}^{k} \gamma_i s_i / p_i}\right)^{\beta_i} > 1 \text{ for all } i = k+1, \ldots, n \quad (A3)$$

That is, for both contributors and noncontributors, public good input assistance results in a larger increase in production. However, if $\gamma_i$ (for $i = k+1, \ldots, n$) is not appropriately small, the private good input assistance is better.
Appendix 2: Data

The annual data are compiled from World Development Indicators provided by the World Bank (http://databank.worldbank.org/data/). We selected the proxy data for each “good or service” from one of the following categories: Infrastructure, Economy & Growth, Private Sector, or Social Protection & Labor.

1. Public good input $g$: Air transport, registered carrier departures worldwide (http://data.worldbank.org/indicator/IS.AIR.DPRT) from the category Infrastructure. Data are measured in terms of registered carrier departures worldwide. In this paper, $g$ is measured in thousands.


4. Labor force $L$: Labor force, total. (http://data.worldbank.org/indicator/SL.TLF.TOTL.IN). Data are measured in terms of number of persons. In this paper, $L$ is measured in millions.

Note that the choice of proxy variables for private good inputs and public good inputs is controversial. Foreign direct investment generally refers to building new facilities as a private good input. In this sense, it is comparable to the public good input, economic infrastructure. We can choose private good inputs from the category “Private Sector”, although this category refers mainly to merchandise exports and imports or manufacturing exports and imports. It is not clear whether such private good inputs should be assisted or not. However, when we choose “High-technology exports (current US$)” as a proxy variable for private good inputs, its choice is not appropriate because the “high-technology exports” input is not generally used as an input into production in recipients (developing) countries. Total employment is a better measure than total labor force; however, appropriate employment data do not exist in this data set. When we have tried to use either rail lines (total route in km), paved roads (% of total roads), or air transport (registered carrier departures worldwide) for public good input $g$, these data are available for only a few countries and a few periods. However, in this paper we use airports, which are indispensable economic infrastructure for any country, and thus air transport data are more available than the other data.

1 The other data for rail lines (total route in km): (http://data.worldbank.org/indicator/IS.RRS.TOTL.KM/countries) and for roads, paved (% of total roads): (http://data.worldbank.org/indicator/IS.ROD.PAVE.ZS/countries) are available for only a few countries.

2 The data for external debt stocks, private nonguaranteed (PNG) (DOD, current US$) are available for only a few countries: http://data.worldbank.org/indicator/DT.DOD.DPNG.CD/countries.

3 The data are from http://data.worldbank.org/indicator/TX.VAL.TECH.CD/countries.
Appendix 3: Estimated $\beta$ including total public good $G$.

We assume that MA, INA, TH, and PH are positive contributors for all periods and estimate the following:

$$\log Y_t = \log A + \delta_1 DU1 + \delta_2 DU2 + \alpha \log x_t + \beta \log G_t : A, a, \beta > 0, \alpha + \beta = 1$$  \hspace{1cm} (A4)

where $G_t = \sum_{i=1}^{4} g_{i,t}$, $i$ is MA, INA, TH, and PH. As seen by comparing Tables 1 and 4, the estimates of $a$ and $\beta$ for PH have changed substantially, while those for other countries is changed only slightly because most of the large changes are absorbed in the coefficients of the dummy variables and a constant term. As in Table 3, we can identify contributors and free riders. Even when we use the new estimates in Table 4, we find that MA, INA, TH, and PH are contributors and the new equilibrium is $G^*_k$ (=844,885) which is close to $G^*_k$ (=936,708) in Table 3.