Supply, Demand and Monetary Policy Shocks in a Multi-Country New Keynesian model

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Grateful to ECB for support, but views not necessarily those of the ECB

DSGE based New Keynesian Models 1

- There exists a large empirical literature on NK-DSGE models - Christiano and Eichenbaum; Smets and Wouters; Lubik and Schorfheide, and others - derived from FOC of representative agents, with rational expectations and staggered pricing
- Yields non-linear stochastic RE equations that are typically log-linearised around the steady state solution, assumed to be stable and unique
- Appropriate if the log-linearisation is carried out around the correct steady state, and the approximation errors are relatively small
- Not appropriate for the analysis of large shocks, and/or when aggregation over heterogeneous agents is important or where expectations are not rational

DSGE based New Keynesian Models 2

- Variables are measured as deviations from the steady states
- Typical structure has Phillips Curve (PC), determining inflation; IS curve determining output and Taylor Rule (TR) type equation determining short term interest rates
- Identification issues, see Canova and Sala, JME 2009, DPSS JMCB 2009, etc.
- Choice of estimation and inference procedures: GMM, ML, and Bayesian methods
- Models used to examine the effects of identified shocks, although there is less agreement about the nature (naming) of the shocks

Open Economy Aspects

- Less work on multi-country versions of NK model. Existing work is largely two country, e.g. Svensson (2000) such as Australia and US in Buncic & Melecky (Ec Record 2008) or small open economy, SOE, and rest of world, RoW.
- Gali and Monacelli, REStud 2005, provide theoretical SOE model, estimated for four SOEs by Lubik and Schorfheide (L&S) (JME 2007), who have terms of trade in PC, terms of trade and foreign output in IS and exchange rate in TR.
- Fukac and Pagan (JAE 2010) criticise L&S model as dynamically misspecified (no lags). Also L&S do not allow for interactions across the four SOE models they consider.

Objective

- Build a multi-country version of the standard NK model (MCNK Model) which allows for both direct linkages through foreign variables and indirect linkages through error spill over effects.
- The multi-country nature of the exercise raises a range of issues that existing DSGE models which use a two block structure or a small open economy assumption do not face.
- Modelling a large number of countries requires methodological innovations in identification, estimation and solution and in the treatment of exchange rates, the nominal anchor and the covariance matrix used to calculate impulse response functions.

- Variables considered y_{it} log output, π_{it} inflation, r_{it} interest rates, re_{it} real effective exchange.
- All variables are measured as deviations from steady state, e.g. $\widetilde{y}_{it} = y_{it} - y^P_{it}$, constructed as long-horizon forecasts from cointegrating GVAR (which is a reduced form of the global RE system and embodies cointegrating relations). This ensures the deviations to be used in the MCNK model are stationary.
- Estimated for 33 countries 1979Q1-2006Q4, by inequality constrained IV subject to theory restrictions, using lagged values and global variables as instruments, e.g. $\widetilde{y}^*_{it} = \Sigma^N_{j=0} w_{ij} \widetilde{y}_{jt}$.
- Solve the resultant multi-country RE model for all countries simultaneously, providing estimates of structural shocks: supply, demand and monetary policy.
- Identify supply, demand and monetary policy shocks (assumed uncorrelated within each country, but possibly correlated across countries).

- Report IRFs for a US monetary policy shock, a global supply shock and a global demand shock, including bootstrapped standard error bands.
- Report FEVDs for the contribution of the different types of shocks to the variance of output, inflation and interest rates.
- Compare results with the standard de-trending approach using HP filter for output and constant steady states for other variables.
- Examine the effect of cutting off inter-country linkages.

Country specific models 1

For countries $i = 0, 1, ..., N$

$$\widetilde{\pi}_{it} = \beta_{ib}\widetilde{\pi}_{i,t-1} + \beta_{if}E_{t-1}\widetilde{\pi}_{i,t+1} + \beta_{iy}\widetilde{y}_{it} + \varepsilon_{i,st}, \tag{1}$$

$$\widetilde{y}_{it} = \alpha_{ib}\widetilde{y}_{i,t-1} + \alpha_{ir}\left[\widetilde{r}_{it} - E_{t-1}\left(\widetilde{\pi}_{i,t+1}\right)\right] + \alpha_{ie}\widetilde{re}_{it} + \alpha_{iy*}\widetilde{y}^{*}_{it} + \varepsilon_{i,dt}, \tag{2}$$

$$\widetilde{r}_{it} = \gamma_{ib}\widetilde{r}_{i,t-1} + \gamma_{i\pi}\widetilde{\pi}_{it} + \gamma_{iy}\widetilde{y}_{it} + \varepsilon_{i,mt}, \tag{3}$$

and

$$\widetilde{re}_{it} = \rho_i\widetilde{re}_{i,t-1} + \varepsilon_{i,et}, \ \ |\rho_i| < 1, \ i = 1, 2, ..., N. \tag{4}$$

where $\widetilde{y}^{*}_{it} = \Sigma_{j=0}^{N} w_{ij}\widetilde{y}_{jt}$ and $\widetilde{re}_{it} = \widetilde{ep}_{it} - \widetilde{ep}^{*}_{it}$, $ep_{it} = e_{it} - p_{it}$.

No intercepts: all variables are in deviations from their associated steady states.

Country specific models 2

- Shocks: as is standard $\varepsilon_{i,st}$ is interpreted as a supply or cost shock, $\varepsilon_{i,dt}$ a demand shock, $\varepsilon_{i,mt}$ a monetary policy shock.
- $\varepsilon_{i,et}$ is a reduced form exchange rate shock which may be correlated with all the other shocks.
- Tried future output in IS curve, but the results were either insignificant or not sensible. Fuhrer and Rudebusch (2004) also find no evidence for future output in IS equation for the US.
- Real effective exchange rates are reduced form AR(1) processes, and the UIP risk premium $r_{it} - r_{it}^{*} - E_{t-1}(e_{i,t+1} - e_{it})$ is determined endogenously.
- Each equation estimated by IV, subject to (theory-based) sign restrictions, instruments are: intercept, $\widetilde{y}_{i,t-1}$, $\widetilde{\pi}_{i,t-1}$, $\widetilde{r}_{i,t-1}$, $\widetilde{re}_{i,t-1}$, $\widetilde{y}_{it}^{*}$, $\widetilde{\pi}_{it}^{*}$, $\widetilde{r}_{it}^{*}$, $\widetilde{p}_{t}^{o}$.

Linking country models 1

- For countries $i = 0, 1, ..., N$, let $\widetilde{\mathbf{x}}_{it} = (\widetilde{\pi}_{it}, \widetilde{y}_{it}, \widetilde{r}_{it}, \widetilde{ep}_{it})'$ and global $(k+1) \times 1$ vector $\widetilde{\mathbf{x}}_t = (\widetilde{\mathbf{x}}'_{0t}, \widetilde{\mathbf{x}}'_{1t}, ..., \widetilde{\mathbf{x}}'_{Nt})'$.
- $\widetilde{\mathbf{x}}_{0t}$ includes the redundant US real exchange rate variable.
- Although in the US model $\widetilde{ep}_{0t} = -\widetilde{p}_{0t}$, derived from PC for $\widetilde{\pi}_{0t}$, $\widetilde{ep}_{0t}$ is still needed for the construction of $\widetilde{ep}^*_{it}$, $i = 0, 1, ...N$ that enter the IS equations.
- In terms of $\widetilde{\mathbf{x}}_{it}$ the country-specific models for $i = 0, 1, ..., N$, can be written as

$$\mathbf{A}_{i0}\widetilde{\mathbf{x}}_{it} = \mathbf{A}_{i1}\widetilde{\mathbf{x}}_{i,t-1} + \mathbf{A}_{i2}E_{t-1}(\widetilde{\mathbf{x}}_{i,t+1}) + \mathbf{A}_{i3}\widetilde{\mathbf{x}}^*_{it} + \mathbf{A}_{i4}\widetilde{\mathbf{x}}^*_{i,t-1} + \varepsilon_{it}, \quad (5)$$

where $\widetilde{\mathbf{x}}^*_{it} = (\widetilde{y}^*_{it}, \widetilde{ep}^*_{it})'$.The expectations are taken with respect to a common global information set formed as the union intersection of the individual country information sets, $\mathfrak{I}_{i,t-1}$.

Let $\widetilde{\mathbf{z}}_{it} = (\widetilde{\mathbf{x}}'_{it}, \widetilde{\mathbf{x}}^{*\prime}_{it})'$ then the $N+1$ models specified by (5) can be written compactly as

$$\mathbf{A}_{iz0}\widetilde{\mathbf{z}}_{it} = \mathbf{A}_{iz1}\widetilde{\mathbf{z}}_{i,t-1} + \mathbf{A}_{iz2}E_{t-1}\left(\widetilde{\mathbf{z}}_{i,t+1}\right) + \varepsilon_{it}, \text{ for } i = 0, 1, ..., N. \tag{6}$$

The variables $\widetilde{\mathbf{z}}_{it}$ are linked to the variables in the global model, $\widetilde{\mathbf{x}}_t$, through the identity

$$\widetilde{\mathbf{z}}_{it} = \mathbf{W}_i\widetilde{\mathbf{x}}_t, \tag{7}$$

where the 'link' matrices $\mathbf{W}_i$, $i = 0, 1, ..., N$ are defined in terms of the weights w_{ij}.

Substituting (7) in (6) now yields

$$\mathbf{A}_{iz0}\mathbf{W}_i\widetilde{\mathbf{x}}_t = \mathbf{A}_{iz1}\mathbf{W}_i\widetilde{\mathbf{x}}_{t-1} + \mathbf{A}_{iz2}\mathbf{W}_iE_{t-1}\left(\widetilde{\mathbf{x}}_{t+1}\right) + \varepsilon_{it},\ i = 0, 1, ..., N,$$

and then stacking all the $N+1$ country models we obtain the multi-country RE model for $\widetilde{\mathbf{x}}_t$ as

$$\mathbf{A}_0\widetilde{\mathbf{x}}_t = \mathbf{A}_1\widetilde{\mathbf{x}}_{t-1} + \mathbf{A}_2E_{t-1}\left(\widetilde{\mathbf{x}}_{t+1}\right) + \varepsilon_t, \tag{8}$$

where the $\mathbf{A}_j, j = 0, 1, 2$ are stacked $k \times (k+1)$ matrices.

The multi-country RE model given by (8) represents a system of k variables in $k+1$ RE equations, and as noted above, contains a redundant equation in the US model as specified.

To deal with this redundancy we consider the *new* $k \times 1$ vector $\overset{\circ}{\widetilde{\mathbf{x}}}_t$, where $\overset{\circ}{\widetilde{\mathbf{x}}}_{0t} = (y_{0t}, r_{0t}, ep_{0t})'$ and $\overset{\circ}{\widetilde{\mathbf{x}}}_{it} = \widetilde{\mathbf{x}}_{it}$ for $i = 1, 2, ..., N$.
Using appropriate matrices, for the US we can relate the 4×1 vector $\widetilde{\mathbf{x}}_{0t}$ to the 3×1 vector $\overset{\circ}{\widetilde{\mathbf{x}}}_{0t}$ by

$$\widetilde{\mathbf{x}}_{0t} = \mathbf{S}_{00}\overset{\circ}{\widetilde{\mathbf{x}}}_{0t} - \mathbf{S}_{01}\overset{\circ}{\widetilde{\mathbf{x}}}_{0,t-1}.$$

Similarly, the $(k+1) \times 1$ vector $\widetilde{\mathbf{x}}_t$ can be related to the $k \times 1$ global vector $\overset{\circ}{\widetilde{\mathbf{x}}}_{0t}$ by

$$\widetilde{\mathbf{x}}_t = \mathbf{S}_0\overset{\circ}{\widetilde{\mathbf{x}}}_t - \mathbf{S}_1\overset{\circ}{\widetilde{\mathbf{x}}}_{t-1}. \tag{9}$$

Using (9) in (8) we have

$$\mathbf{H}_0\overset{\circ}{\widetilde{\mathbf{x}}}_t = \mathbf{H}_1\overset{\circ}{\widetilde{\mathbf{x}}}_{t-1} + \mathbf{H}_2\overset{\circ}{\widetilde{\mathbf{x}}}_{t-2} + \mathbf{H}_3E_{t-1}(\overset{\circ}{\widetilde{\mathbf{x}}}_{t+1}) + \mathbf{H}_4E_{t-1}(\overset{\circ}{\widetilde{\mathbf{x}}}_t) + \varepsilon_t. \tag{10}$$

For a determinate solution the $k \times k$ matrix $\mathbf{H}_0$ must be non-singular.

Pre-multiplying (10) by $\mathbf{H}_0^{-1}$

$$\overset{\circ}{\widetilde{\mathbf{x}}}_t = \mathbf{F}_1 \overset{\circ}{\widetilde{\mathbf{x}}}_{t-1} + \mathbf{F}_2 \overset{\circ}{\widetilde{\mathbf{x}}}_{t-2} + \mathbf{F}_3 E_{t-1}(\overset{\circ}{\widetilde{\mathbf{x}}}_{t+1}) + \mathbf{F}_4 E_{t-1}(\overset{\circ}{\widetilde{\mathbf{x}}}_t) + \mathbf{u}_t, \qquad (11)$$

where $\mathbf{F}_j = \mathbf{H}_0^{-1}\mathbf{H}_j$, for $j = 1, 2, 3, 4$, and $\mathbf{u}_t = \mathbf{H}_0^{-1}\varepsilon_t$.

Using a companion form representation (11) can be written as the canonical RE model (Binder and Pesaran, 1995)

$$\chi_t = \mathbf{A}\chi_{t-1} + \mathbf{B}E_{t-1}(\chi_{t+1}) + \eta_t, \qquad (12)$$

where $\chi_t = \left(\overset{\circ}{\widetilde{\mathbf{x}}}'_t, \overset{\circ}{\widetilde{\mathbf{x}}}'_{t-1}\right)'$, and

$$\mathbf{A} = \begin{pmatrix} \mathbf{F}_1 & \mathbf{F}_2 \\ \mathbf{I}_k & \mathbf{0} \end{pmatrix}, \ \mathbf{B} = \begin{pmatrix} \mathbf{F}_3 & \mathbf{F}_4 \\ \mathbf{0} & \mathbf{0} \end{pmatrix}, \ \eta_t = \begin{pmatrix} \mathbf{u}_t \\ \mathbf{0} \end{pmatrix}.$$

The solution properties of this system, discussed in Binder and Pesaran (1995, 1997), depends on the roots of the quadratic matrix equation

$$\mathbf{B}\Phi^2 - \Phi + \mathbf{A} = \mathbf{0}. \tag{13}$$

There will be a unique globally consistent stationary solution if (13) has a real matrix solution such that all the eigenvalues of Φ and $(\mathbf{I} - \mathbf{B}\Phi)^{-1}\boldsymbol{B}$ lie strictly inside the unit circle.

The solution is then given by

$$\boldsymbol{\chi}_t = \Phi\boldsymbol{\chi}_{t-1} + \boldsymbol{\eta}_t. \tag{14}$$

Partitioning Φ conformably to χ_t, (14) can be expressed as

$$\begin{pmatrix} \widetilde{\mathring{\mathbf{x}}}_t \\ \widetilde{\mathring{\mathbf{x}}}_{t-1} \end{pmatrix} = \begin{pmatrix} \Phi_{11} & \Phi_{12} \\ \mathbf{I}_k & \mathbf{0} \end{pmatrix} \begin{pmatrix} \widetilde{\mathring{\mathbf{x}}}_{t-1} \\ \widetilde{\mathring{\mathbf{x}}}_{t-2} \end{pmatrix} + \begin{pmatrix} \mathbf{I}_k & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_k \end{pmatrix} \begin{pmatrix} \mathbf{u}_t \\ \mathbf{0} \end{pmatrix},$$

so that the solution in terms of $\widetilde{\mathring{\mathbf{x}}}_t$, is given by

$$\widetilde{\mathring{\mathbf{x}}}_t = \Phi_{11}\widetilde{\mathring{\mathbf{x}}}_{t-1} + \Phi_{12}\widetilde{\mathring{\mathbf{x}}}_{t-2} + \mathbf{H}_0^{-1}\boldsymbol{\varepsilon}_t, \tag{15}$$

where $\boldsymbol{\varepsilon}_t = (\boldsymbol{\varepsilon}'_{0t}, \boldsymbol{\varepsilon}'_{1t}, ..., \boldsymbol{\varepsilon}'_{Nt})'$.

The structural shocks, $\boldsymbol{\varepsilon}_t$, can be recovered by noting that

$$\boldsymbol{\varepsilon}_t = \mathbf{H}_0(\widetilde{\mathring{\mathbf{x}}}_t - \Phi_{11}\widetilde{\mathring{\mathbf{x}}}_{t-1} - \Phi_{12}\widetilde{\mathring{\mathbf{x}}}_{t-2}). \tag{16}$$

The covariance matrix of the structural shocks is given by

$$E(\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}'_t) = \Sigma_{\varepsilon}, \tag{17}$$

which can be obtained from the estimated structural shocks.

It will be convenient to reorder the elements of ε_t in (16) in terms of the different types of shocks as $\varepsilon_t^0 = (\varepsilon'_{st}, \varepsilon'_{dt}, \varepsilon'_{mt}, \varepsilon'_{et})'$, where

- ε_{st} and ε_{dt} are the $(N+1) \times 1$ vectors of supply and demand shocks
- ε_{mt} and ε_{et} are the $N \times 1$ vectors of monetary policy shocks (for all countries except Saudi Arabia) and shocks to the real effective exchange rates (for all countries except the US).

We can then write

$$\varepsilon_t^0 = \mathbf{G}\varepsilon_t, \quad (18)$$

where $\mathbf{G}$ is a non-singular $k \times k$ matrix with elements 0 or 1.

Also $E(\varepsilon_t^0 \varepsilon_t^{0\prime}) = \Sigma_\varepsilon^0 = \mathbf{G}\Sigma_{\boldsymbol{\varepsilon}}\mathbf{G}'$, which can be obtained from Σ_ε by suitable permutations of its rows and columns.

Covariance Matrix

Assume no correlation between different types of structural shocks (supply, demand and monetary policy), though the same type of structural shocks in different countries can be correlated; and Exchange rate shocks can have non-zero correlations with the other shocks both within and across countries. This gives a bordered block diagonal error covariance matrix,

$$\Sigma_{\varepsilon}^{0} = \begin{pmatrix} \Sigma_{ss} & \mathbf{0} & \mathbf{0} & \Sigma_{se} \\ \mathbf{0} & \Sigma_{dd} & \mathbf{0} & \Sigma_{de} \\ \mathbf{0} & \mathbf{0} & \Sigma_{mm} & \Sigma_{me} \\ \Sigma_{es} & \Sigma_{ed} & \Sigma_{em} & \Sigma_{ee} \end{pmatrix}. \tag{19}$$

Σ_{ss} and Σ_{dd} : $(N+1) \times (N+1)$ covariance matrices of supply and demand shocks
Σ_{mm} and Σ_{ee} : $N \times N$ covariance matrices of the monetary policy and exchange rate shocks
Σ_{es} : covariances between the exchange rate and supply shocks etc

Shock accounting: IRFs and FEVDs

Consider the effect of a particular shock, $\xi_t = \mathbf{a}'\varepsilon_t^0$ on a composite variable $q_t = \mathbf{b}'\widetilde{\mathbf{x}}_t$.

The $k \times 1$ vector **a** and the $(k+1) \times 1$ vector **b** are either selection vectors or weighting vectors to give e.g. a global supply shock, or a PPP GDP weighted averages of the variables for the eight euro area countries.

The IRFs provide the time profile of the response by $q_t = \mathbf{b}'\widetilde{\mathbf{x}}_t$ to a unit shock to $\xi_t = \mathbf{a}'\varepsilon_t^0$.

The FEVDs estimate the relative importance of different shocks in explaining the variations in output, inflation and interest rates from their steady states in a particular economy over time.

Deviations from steady state

Fukac and Pagan (JAE 2009) discuss alternative methods of taking deviations.

The global model is specified in terms of the realised values denoted by $\mathbf{x}_t = (\mathbf{x}'_{0t}, \mathbf{x}'_{1t}, ..., \mathbf{x}'_{Nt})'$, with the deviations given by

$$\widetilde{\mathbf{x}}_t = \mathbf{x}_t - \mathbf{x}_t^P,$$

where $\mathbf{x}_t^P$ denotes the permanent component of $\mathbf{x}_t$.

$\mathbf{x}_t^P$ is further decomposed into deterministic and stochastic components

$$\mathbf{x}_t^P = \mathbf{x}_{d,t}^P + \mathbf{x}_{s,t}^P, \text{ and } \mathbf{x}_{d,t}^P = \boldsymbol{\mu} + \mathbf{g}t,$$

where $\boldsymbol{\mu}$ and $\mathbf{g}$ are $k \times 1$ vectors of constants and t a deterministic time trend.

Long horizon forecasts

The steady state (permanent-stochastic component) $\mathbf{x}^P_{st}$, is then defined as the 'long-horizon forecast' (net of the permanent-deterministic component)

$$\mathbf{x}^P_{s,t} = \lim_{h\to\infty} E_t\left(\mathbf{x}_{t+h} - \mathbf{x}^P_{d,t+h}\right) = \lim_{h\to\infty} E_t\left[\mathbf{x}_{t+h} - \boldsymbol{\mu} - \mathbf{g}(t+h)\right].$$

- Multivariate Beveridge-Nelson type decomposition.
- Estimates of the permanent components can be based on a GVAR (a reduced form version of the MCNK RE solution), taking account of unit roots and cointegration in the global economy.
- Uniquely define deviations from steady states $\widetilde{\mathbf{x}}_t = \mathbf{x}_t - \mathbf{x}^P_t$.
- There is no need to identify stochastic trends.

Estimation method

- Each equation is estimated by inequality constrained IV over the period of 1980Q1-2006Q4 (except for Argentina's PC equation)
- Instruments: intercept, $\widetilde{\pi}_{i,t-1}, \widetilde{y}_{i,t-1}, \widetilde{r}_{i,t-1}, \widetilde{re}_{i,t-1}, \widetilde{\pi}^*_{it}, \widetilde{y}^*_{it}, \widetilde{r}^*_{it}, \widetilde{p}^o_t$ for all countries but Saudi Arabia, where $\widetilde{r}_{i,t-1}$ and $\widetilde{r}^*_{it}$ are excluded
- The inequality constraints are motivated by theory and to avoid non-uniqueness of the solution
- Bayesian methods difficult to implement on a system of this size

Phillips Curve:

The unrestricted model, PC_u, is

$$\widetilde{\pi}_{it} = \beta_{ib}\widetilde{\pi}_{i,t-1} + \beta_{if}E_{t-1}\widetilde{\pi}_{i,t+1} + \beta_{iy}\widetilde{y}_{it} + \varepsilon_{i,st}.$$

- The parameters of the Phillips curve are estimated subject to the inequality restrictions $\beta_{ib} \geq 0$, $\beta_{if} \geq 0$, $\beta_{ib} + \beta_{if} \leq 0.99$, and $\beta_{iy} \geq 0$. Since under $\beta_{ib} = \beta_{if} = 0$, the third restriction, $\beta_{ib} + \beta_{if} \leq 0.99$, is satisfied, there are 14 possible specifications.
- All specifications are estimated and from those satisfying the restrictions the one with the lowest in-sample mean squared prediction error is selected.
- In the case of 7 countries, the IV estimates satisfied all the constraints. Also the coefficient of inflation expectations, β_{if}, turned out to be positive in all cases; and generally much larger than the coefficient of lagged inflation, β_{ib}.

Distribution of inequality-constrained IV estimates

	Mean	# Constrained	UC Mean	Constraint
Phillips curve, N=33				
β_{ib}	0.12	10	0.17	$\beta_{ib} \geq 0$
β_{if}	0.80	0	0.80	$\beta_{if} \geq 0$
β_{iy}	0.11	7	0.14	$\beta_{iy} \geq 0$
$\beta_{ib}+\beta_{if}$	0.93	22	0.80	$\beta_{ib}+\beta_{if} \leq 0.99$

IS Curve:
The unrestricted model IS_u is

$$\widetilde{y}_{it} = \alpha_{ib}\widetilde{y}_{i,t-1} + \alpha_{ir}[\widetilde{r}_{it} - E_{t-1}\left(\widetilde{\pi}_{i,t+1}\right)] + \alpha_{ie}\widetilde{re}_{it} + \alpha_{iy*}\widetilde{y}^{*}_{it} + \varepsilon_{i,dt}.$$

- We opted for the IS specification without the future output variable, and estimated the parameters subject to the constraints $\alpha_{ir} \leq 0$ and $\alpha_{iy*} \geq 0$, following the same procedure as before.
- The unrestricted equation was chosen for 14 countries.
- Including $\widetilde{y}^{*}_{it}$ tended to produce a more negative and significant estimate of the interest rate effect, and in the case of the US the estimate was positive unless $\widetilde{y}^{*}_{it}$ was included.
- The estimate of the coefficient of the real exchange rate variable averaged to about zero, but with quite a large range of variations across the different countries.

Distribution of inequality-constrained IV estimates

	Mean	# Constrained	UC Mean	Constraint
IS curve, N=33				
α_{ib}	0.27	0	0.27	
α_{ir}	-0.20	18	-0.43	$\alpha_{ir} \leq 0$
α_{ie}	0.02	0	0.02	
α_{iy*}	0.79	2	0.84	$\alpha_{iy*} \geq 0$

Taylor Rule:

The unrestricted model TR_u is

$$\widetilde{r}_{it} = \gamma_{ir}\widetilde{r}_{i,t-1} + \gamma_{i\pi}\widetilde{\pi}_{it} + \gamma_{iy}\widetilde{y}_{it} + \varepsilon_{i,mt}.$$

- The parameters are estimated subject to the constraints $\gamma_{iy} \geq 0$ and $\gamma_{i\pi} \geq 0$.
- The unrestricted equation was chosen for 18 countries out of the 32 possible Taylor rule equations.
- In the case of Malaysia a fully constrained specification with $\gamma_{iy} = \gamma_{i\pi} = 0$, resulted, and in 3 other countries we obtained the restricted case with $\gamma_{i\pi} = 0$. In 11 countries, including the US, we ended up with $\gamma_{iy} = 0$.

Real effective exchange rate:

- OLS estimates of ρ_i ranged from 0.34 to 0.86, confirming that this is a stable process

Distribution of inequality-constrained IV estimates

	Mean	# Constrained	UC Mean	Constraint
Taylor Rule, N=32				
γ_{ib}	0.59	0	0.59	
$\gamma_{i\pi}$	0.24	4	0.28	$\gamma_{ir} \geq 0$
γ_{iy}	0.06	11	0.09	$\gamma_{iy} \geq 0$
Exchange rates, N=32				
ρ_i	0.67	0	0.67	$\lvert\rho_i\rvert < 1$

Inequality-constrained IV estimates for eight major economies

	US	China	Japan	Germ.	France	UK	Italy	Can.
Phillips curve								
β_{ib}	0.22	0.00	0.11	0.10	0.00	0.12	0.38	0.22
β_{if}	0.77	0.86	0.84	0.89	0.99	0.87	0.61	0.77
β_{iy}	0.10	0.14	0.13	0.04	0.08	0.05	0.04	0.02

Inequality-constrained IV estimates for eight major economies

	US	China	Japan	Germ.	France	UK	Italy	Can.
IS curve								
α_{ib}	0.21	0.72	0.15	0.03	0.00	0.54	0.23	0.37
α_{ir}	-0.98	-0.47	-0.23	0.00	-0.02	0.00	0.00	-1.41
α_{ie}	-0.01	0.02	0.02	-0.04	-0.01	0.25	-0.02	0.12
α_{iy*}	0.74	0.31	0.15	1.10	0.64	0.95	0.73	0.89

Inequality-constrained IV estimates for eight major economies

	US	China	Japan	Germ.	France	UK	Italy	Can.
Taylor Rule								
γ_{ib}	0.79	0.98	0.82	0.62	0.94	0.74	0.82	0.51
$\gamma_{i\pi}$	0.28	0.11	0.21	0.27	0.04	0.20	0.20	0.42
γ_{iy}	0.00	0.00	0.15	0.04	0.03	0.01	0.01	0.00
Exchange rate equation								
ρ_i		0.78	0.76	0.54	0.68	0.53	0.73	0.84

Solution and covariance matrix of the shocks

- The multi-country NK model is solved for all time periods in our estimation sample, and allows us to obtain estimates of all the structural shocks in the model. Altogether there are 130 different shocks; 98 structural and 32 reduced form.
- Denote the shock of type $k = s, d, m, e$ in country $i = 1, 2, ..., 33$ at time $t = 1980Q1 - 2006Q4$ by $\varepsilon_{i,k,t}$. It is now possible to compute pair-wise correlations of any pair of shocks both within and across countries.

Average pair-wise correlations of shocks using GVAR deviations.

	Supply	Demand	Mon. Pol.	Ex. Rate
Supply	0.495	0.166	0.040	0.048
Demand		0.067	0.063	-0.005
Mon. Pol.			0.139	-0.043
Ex. Rate				0.049

US monetary policy shock

- We first consider a contractionary US monetary policy shock, $\mathbf{a}_m' \varepsilon_t^0$, where $\mathbf{a}_m$ has zeros except for the element corresponding to, $\varepsilon_{0,mt}$, which is set to unity.
- The US monetary policy shock raises the US interest rate on impact by one standard error (around 22 basis points per quarter), which also simultaneously impacts interest rates in other countries through the contemporaneous dependence of monetary policy shocks as captured by the off diagonal elements of $\hat{\Sigma}_{mm}$.
- The results for the five Latin American countries (Argentina, Brazil, Chile, Mexico, Peru), Indonesia and Turkey are not included on the graphs as they tend to be outliers due to the much higher levels of inflation and nominal interest rates.
- To focus on the differences across countries, the graphs only show the point estimates, some bootstrap bounds below.

Figure 1a: Impulse responses of a one standard error US monetary policy shock on interest rates (per cent per quarter)

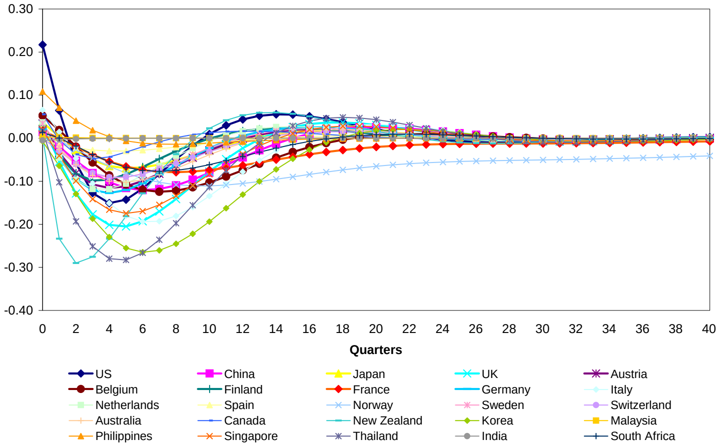


Figure 1b: Impulse responses of a one standard error US monetary policy shock on inflation (per cent per quarter)

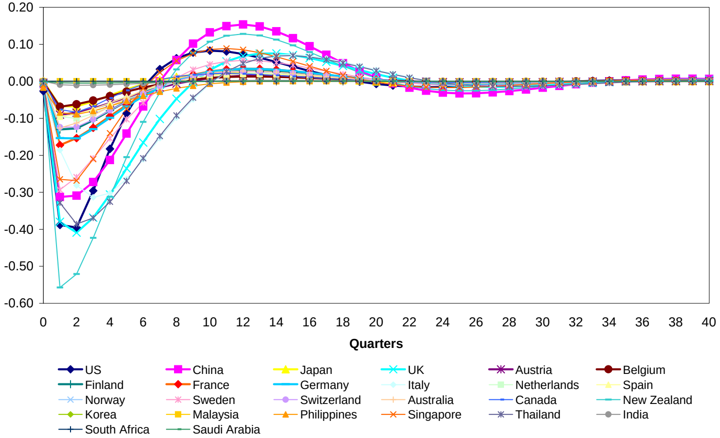
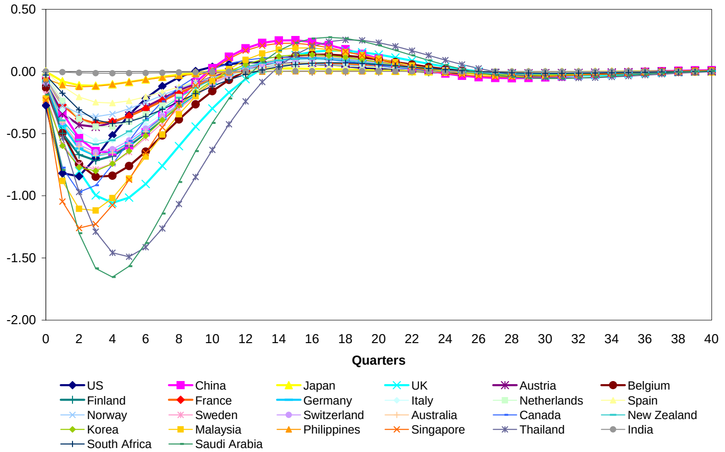


Figure 1c: Impulse responses of a one standard error US monetary policy shock on output (per cent per quarter)



Global supply shock

- We now consider a global inflationary supply shock, $\mathbf{a}_s'\varepsilon_t^0$, where the non-zero elements of $\mathbf{a}_s$ are PPP GDP weights (that add up to one), associated with the $N+1$ supply shocks, $\varepsilon_{i,st}$, in ε_t^0.
- The supply shock causes inflation and interest rates to increase on impact, but then they both fall below their steady state values relatively rapidly, before slowly returning back to the steady states.
- The pattern is similar across other countries, though the impact effect on the US is rather higher than the average increase in inflation experienced in other countries.

Figure 2a: Impulse responses of a one standard error global supply shock on inflation (per cent per quarter)

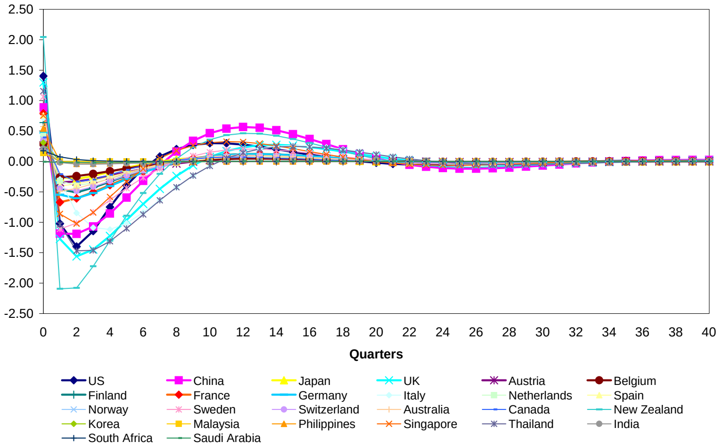


Figure 2b: Impulse responses of a one standard error global supply shock on output (per cent per quarter)

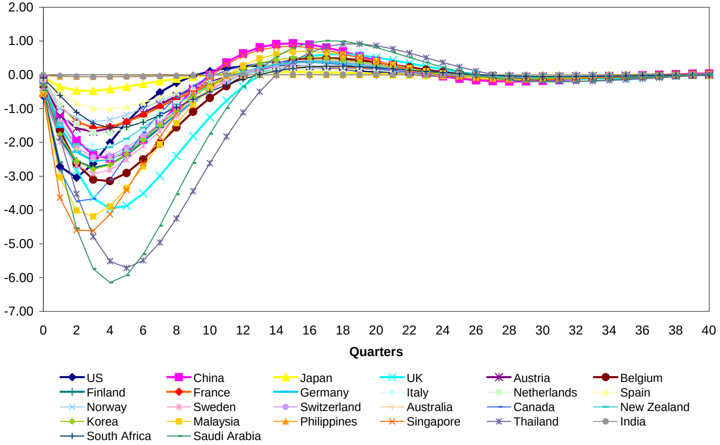
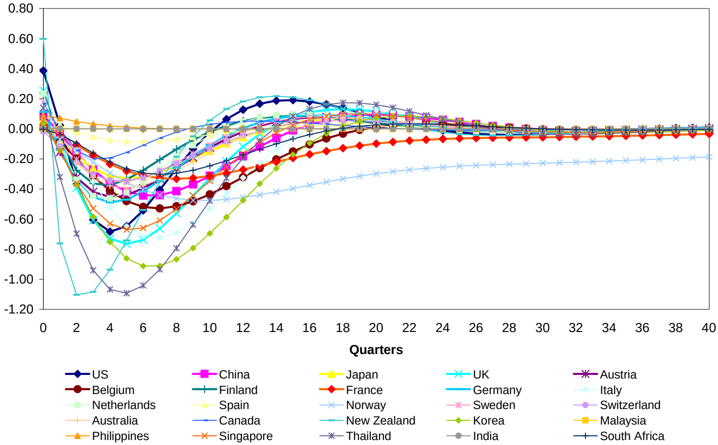


Figure 2c: Impulse responses of a one standard error global supply shock on interest rates (per cent per quarter)



Global demand shock

- The effects of a global demand shock are constructed similarly to the global supply shock using PPP GDP weights.
- As expected the demand shock has a positive effect on output, inflation and interest rates.
- The global demand shock causes output and interest rates to rise before cycling back to their steady state values. The initial expansionary phase of the shock is relatively long lived and takes around 11 to 15 quarters.
- The effects of the demand shock across countries are qualitatively similar, but differ markedly in the size of the effects.
- The positive effect on inflation lasts a somewhat shorter period than on output.

Figure 3a: Impulse responses of a one standard error global demand shock on output (per cent per quarter)

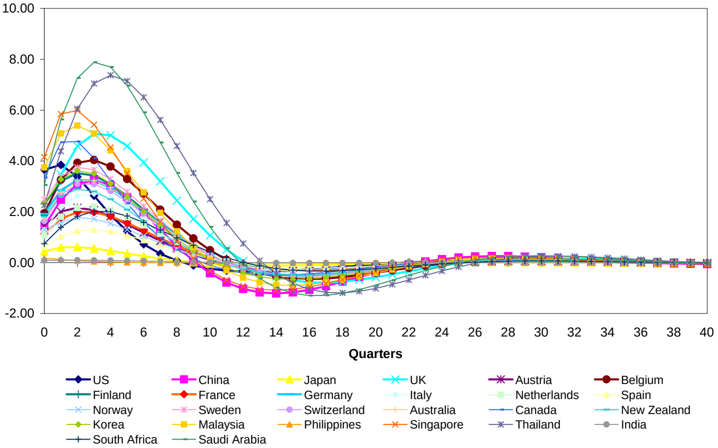


Figure 3b: Impulse responses of a one standard error global demand shock on inflation (per cent per quarter)

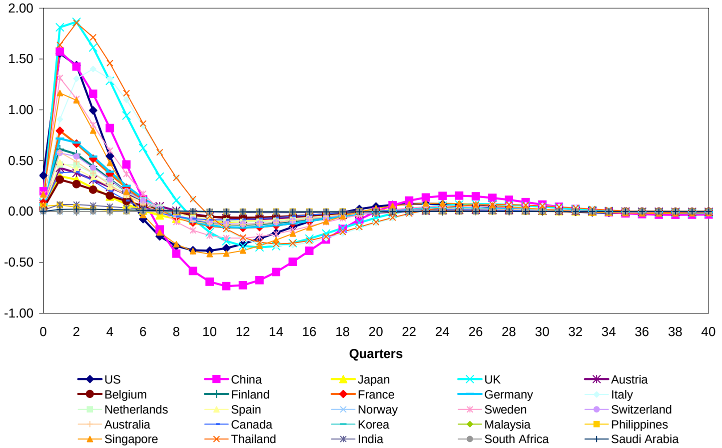
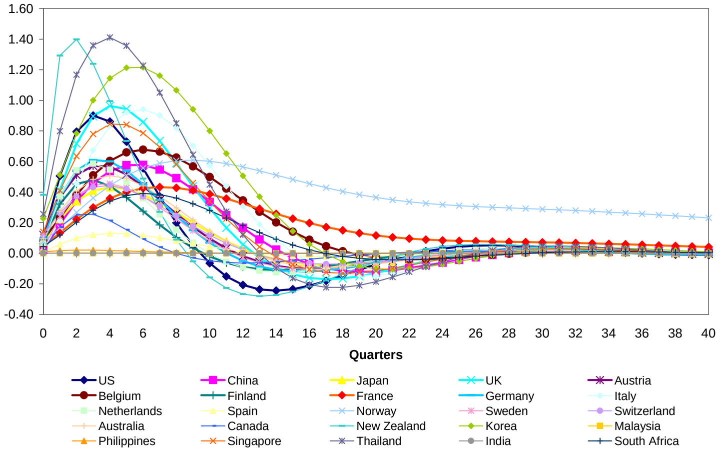


Figure 3c: Impulse responses of a one standard error global demand shock on interest rates (per cent per quarter)



FEVDs

The model was used to estimate the contribution of different shocks to the variations in output, inflation and interest rates.

- Supply and demand shocks account for most of the variation in output, inflation and interest rate in the long-run, with monetary policy shocks and exchange rate shocks accounting for relatively little of the variation.
- Monetary policy shocks account for more of the variation in interest rates in Canada than in other countries, though even here it is not a large proportion.
- On impact supply shocks account for most of the variation of inflation, but this drops rapidly and these shocks only account for about half of the variation of inflation in the long-run.
- On impact demand shocks account for most of the variations in output, but again this figure drops quite rapidly.

Figure 4a: Forecast error variance decomposition of the shocks in explaining inflation, output and interest rates for the US, the euro area and China

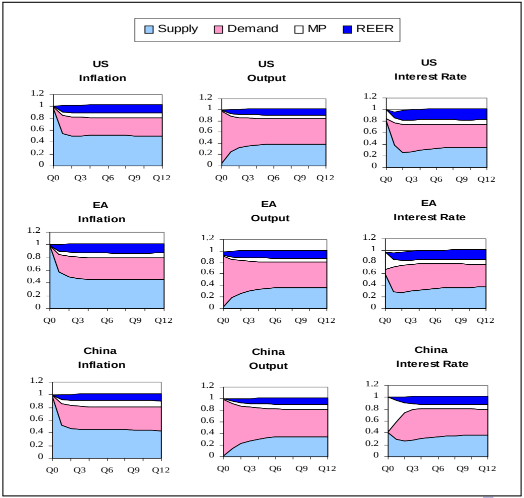
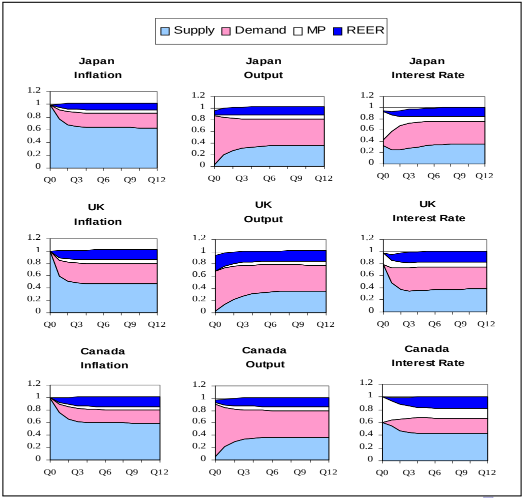


Figure 4b: Forecast error variance decomposition of the shocks in explaining inflation, output and interest rates for Japan, the UK and Canada



Bootstrapped error bands

- We also used a bootstrap procedure to compute 90% error bands for the impulse responses.
- The figures show the median (which is almost identical to the mean except for India, not shown) and the 5% and 95% quantiles of the bootstrap distribution.
- The results indicate that the effects of the shocks are statistically significant in the sense that the 90% bootstrap bands do not always cover zero.

Figure 5a: Impulse responses of a one standard error US monetary policy shock on US and euro area interest rates, inflation and output (per cent per quarter, bootstrap median estimates together with 90% bootstrap bands)

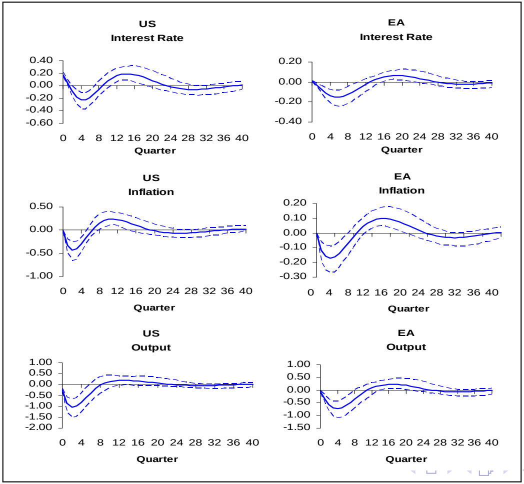


Figure 5b: Impulse responses of a one standard error global supply shock on US and euro area inflation, output and interest rates (per cent per quarter, bootstrap median estimates together with 90% bootstrap bands)

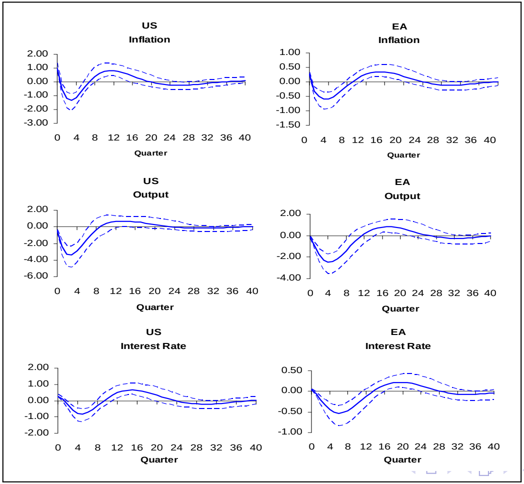
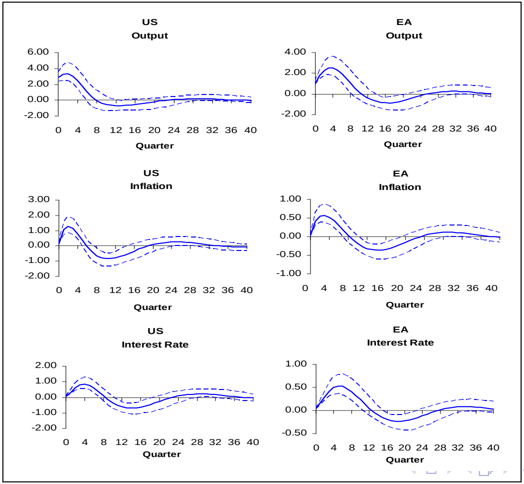


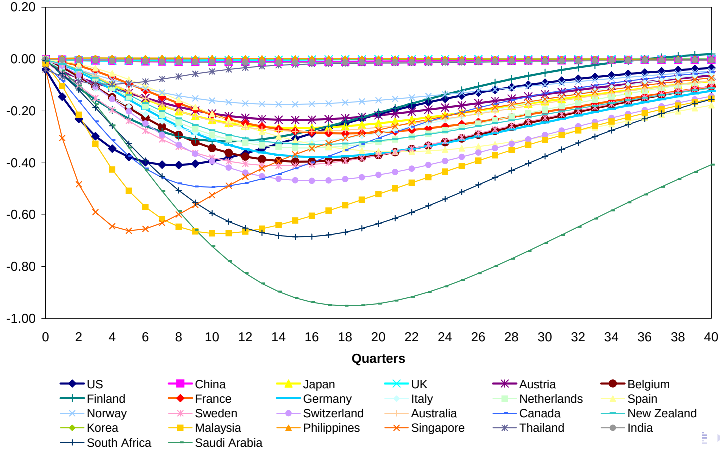
Figure 5c: Impulse responses of a one standard error global demand shock on US and euro area output, inflation and interest rates (per cent per quarter, bootstrap median estimates together with 90% bootstrap bands)



Using HP Filters

- We also used the standard assumption of a HP filter ($\lambda = 1600$) for the output steady state with the steady states of the other variables being constants.
- HP estimates are more backward looking than GVAR estimates, with slower adjustment and near unit roots for $\widetilde{re}_{it}$.
- The effect of $\widetilde{y}_{it}^{HP}$ in the PC is smaller than using $\widetilde{y}_{it}$ from GVAR.
- In the IS curve, in addition to larger lagged effects, domestic output deviations are less responsive to foreign output deviations when using $\widetilde{y}_{it}^{HP}$.

Figure 6: Impulse responses of a one standard error US monetary policy shock on output using HP deviations and constant steady states (per cent per quarter)



Shutting off direct international linkages

- We estimate the MCNK model (with GVAR deviations) setting the coefficient of the foreign output variable in the IS curve to zero.
- This causes the average pair-wise correlation coefficient across the demand shocks to increase from 0.166 to 0.229, thus shifting the burden of the international transmission of shocks to the indirect effects as captured by error covariances.
- The impulse response functions also became much less sensible. In response to a US monetary policy shock, interest rates rise almost everywhere, but the response of output and inflation is much more dispersed as compared to the results from the baseline model.

Figure 7a: Impulse responses of a one standard error US monetary policy shock on interest rates in model without foreign output (per cent per quarter)

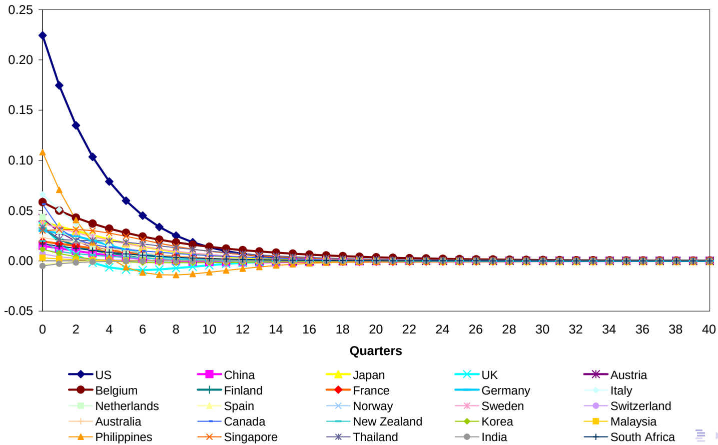


Figure 7b: Impulse responses of a one standard error US monetary policy shock on inflation in model without foreign output (per cent per quarter)

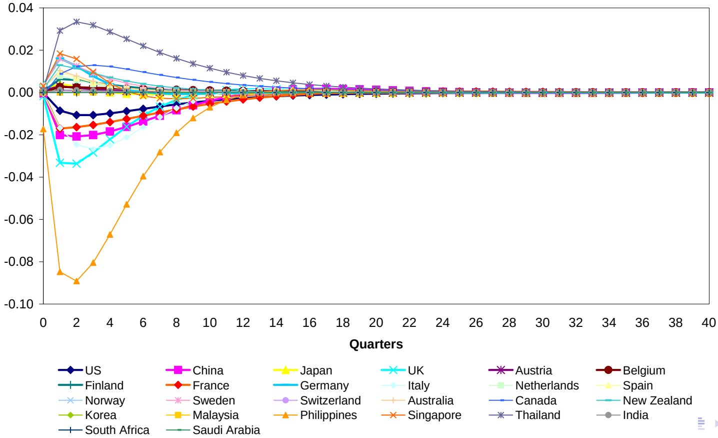
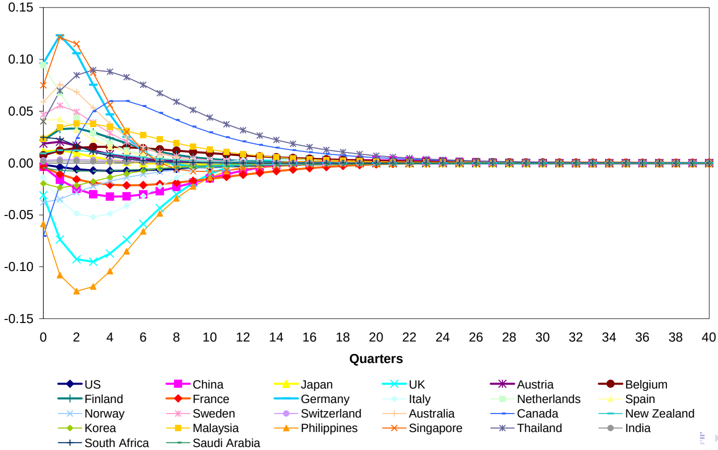


Figure 7c: Impulse responses of a one standard error US monetary policy shock on output in model without foreign output
(per cent per quarter)



Conclusion

- Possible to estimate, solve and simulate a forward-looking multi-country New Keynesian model and use it to estimate the effects of identified supply, demand and monetary policy shocks.
- For all the focus economies, the qualitative effects of demand and supply shocks are as predicted by the theory.
- Monetary policy shocks offset more quickly than is typically obtained in the literature.
- Global supply and demand shocks are the most important drivers of output, inflation and interest rates. By contrast monetary or exchange rate shocks have only a short-run role in the evolution of the world economy.
- Direct channels of transmission of shocks are far more important than indirect transmissions through error spillover effects.

Future developments

- Further developments of the model to
  - allow for financial variables - credits, long term interest rates, real equity prices
  - allow for asset imbalances across economies and their feedbacks (US dollar acting as a store of value and not just a unit of account)
  - introduce international trade variables, such as exports and imports directly rather than indirectly through trade weights.
- The MCNK model developed in this paper provides a natural framework for such extensions.